Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 21 January 2019

Morning (Time: 2 hours) | Paper Reference **4PM0/02**

Further Pure Mathematics
Paper 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

1/1/1/

Pearson

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Solve the equation $3\log_3 x - 8\log_x 3 = 10$

(6)

P 5 5 8 8 5 A 0 2 3 6

Question 1 continued

(Total for Question 1 is 6 marks)

2 (a) Using the axes below, sketch the line with equation

(i) $y + 2x = -5$ (ii) $y = x + 4$

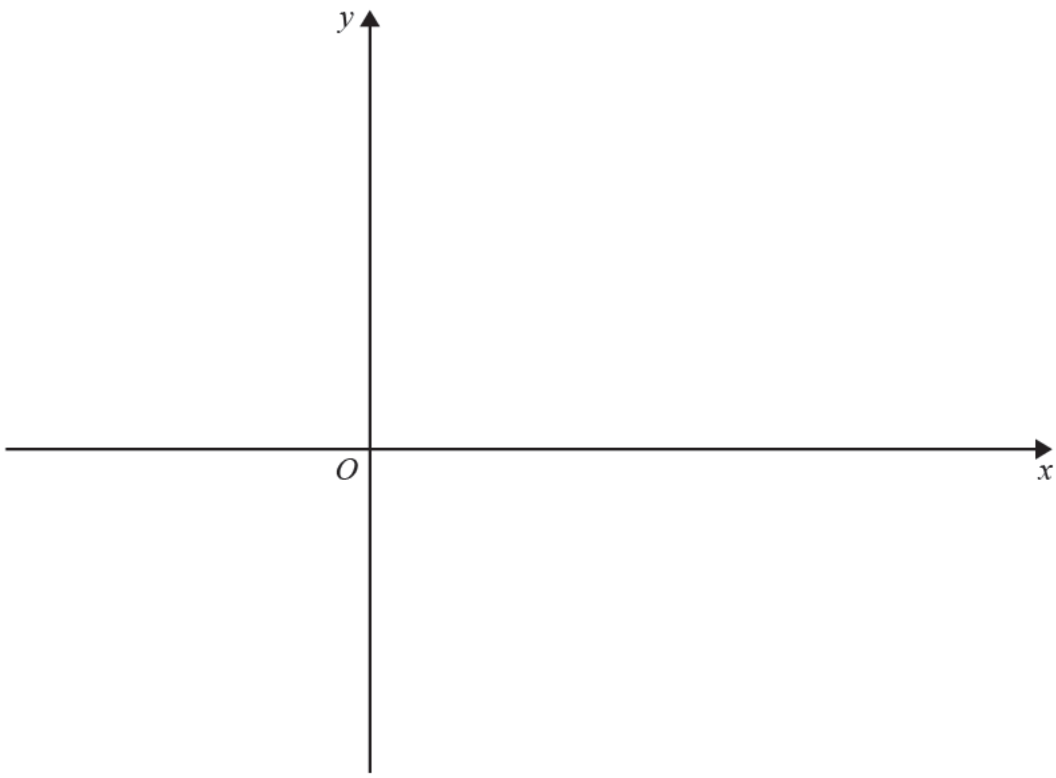
Show the coordinates of the points where each line crosses the coordinate axes.

(2)

(b) Show, by shading, the region R defined by the inequalities

$$y + 2x > -5 \qquad y < x + 4 \qquad x < 1$$

(1)



Question 2 continued

(Total for Question 2 is 3 marks)

3 Referred to a fixed origin O, the position vectors of the points P and Q are $(5\mathbf{i} + 6\mathbf{j})$ and $(3\mathbf{i} - 4\mathbf{j})$ respectively.

(a) Find, as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$, $\overrightarrow{PQ}$.

(2)

(b) Find a unit vector parallel to $\overrightarrow{PQ}$.

(2)

The position vector of the fixed point R is $(13\mathbf{i} + a\mathbf{j})$, where a is a constant.

Given that $\overrightarrow{QR} = 5\overrightarrow{QP}$

(c) find the value of a.

(2)

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Question 3 continued

(Total for Question 3 is 6 marks)

4 A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the velocity, v m/s, of P is given by $v = 4\sin 2t$

(a) Find the least value of t for which the velocity of P is 2 m/s.

(2)

(b) Find the magnitude of the acceleration of P when its velocity is 2 m/s.

(3)

The particle P is at the point with coordinates (3, 0) when $t = \dfrac{\pi}{4}$

(c) Find the distance of P from the origin when $t = 0$

(4)

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Question 4 continued

(Total for Question 4 is 9 marks)

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5

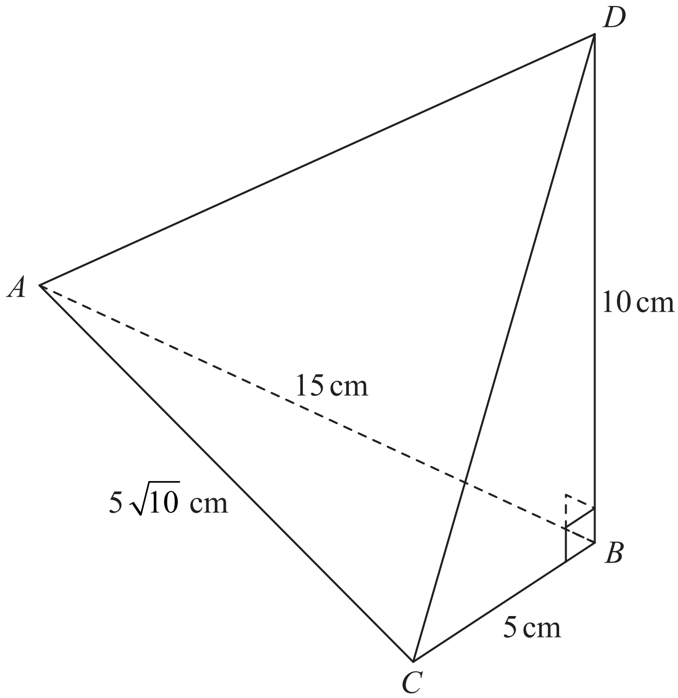


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a triangular pyramid *ABCD* where triangle *ABC* is the base and *BD* is perpendicular to the base.

$$AB = 15\,\text{cm} \quad AC = 5\sqrt{10}\ \text{cm} \quad BC = 5\,\text{cm} \quad BD = 10\,\text{cm}$$

(a) Show that $\angle ABC = 90°$

(2)

(b) Find, in degrees to 1 decimal place, the size of $\angle DAC$.

(4)

The point *X* on *AC* is such that *BX* is perpendicular to *AC*.

(c) Find, in degrees to 1 decimal place, the size of $\angle DXB$.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6

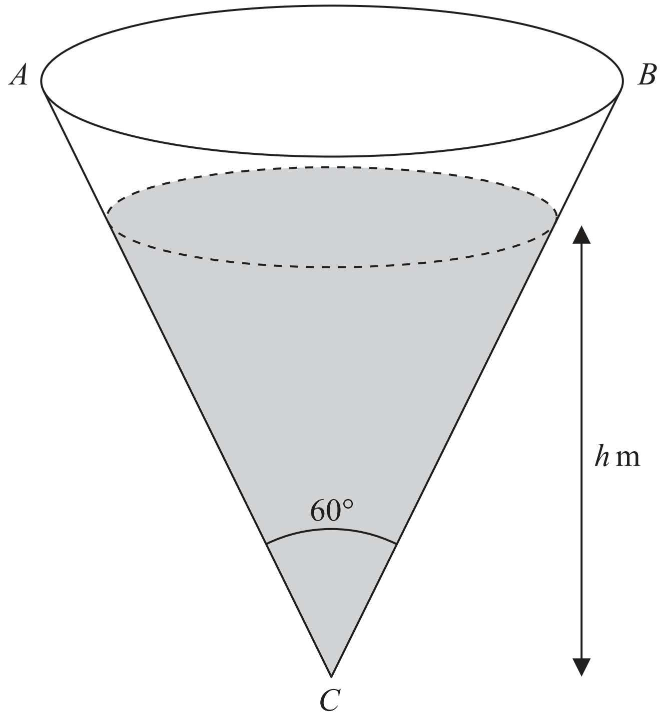


Figure 2

Figure 2 shows a water tank in the shape of a hollow right circular cone fixed with its axis of symmetry vertical. A diameter of the circular rim of the cone is AB. The vertex, C, of the cone is below AB such that $\angle ACB = 60°$

Initially, the tank is empty and water flows into the tank at a constant rate of $0.03\,\text{m}^3/\text{s}$. At time t seconds after the water starts to flow into the tank, the height of the surface of the water in the tank above C is h metres.

Find, in m/s to 3 significant figures, the rate of change of the height of the surface of the water above C at the instant when $h = 1.5$

(6)

Question 6 continued

(Total for Question 6 is 6 marks)

7 (a) Complete the table of values for $y = \ln(3x + 1) + 2$, giving your answers to 2 decimal places.

x	0	1	2	3	4	5	6
y	2		3.95	4.30			4.94

(2)

(b) On the grid opposite, draw the graph of $y = \ln(3x + 1) + 2$ for $0 \leqslant x \leqslant 6$

(2)

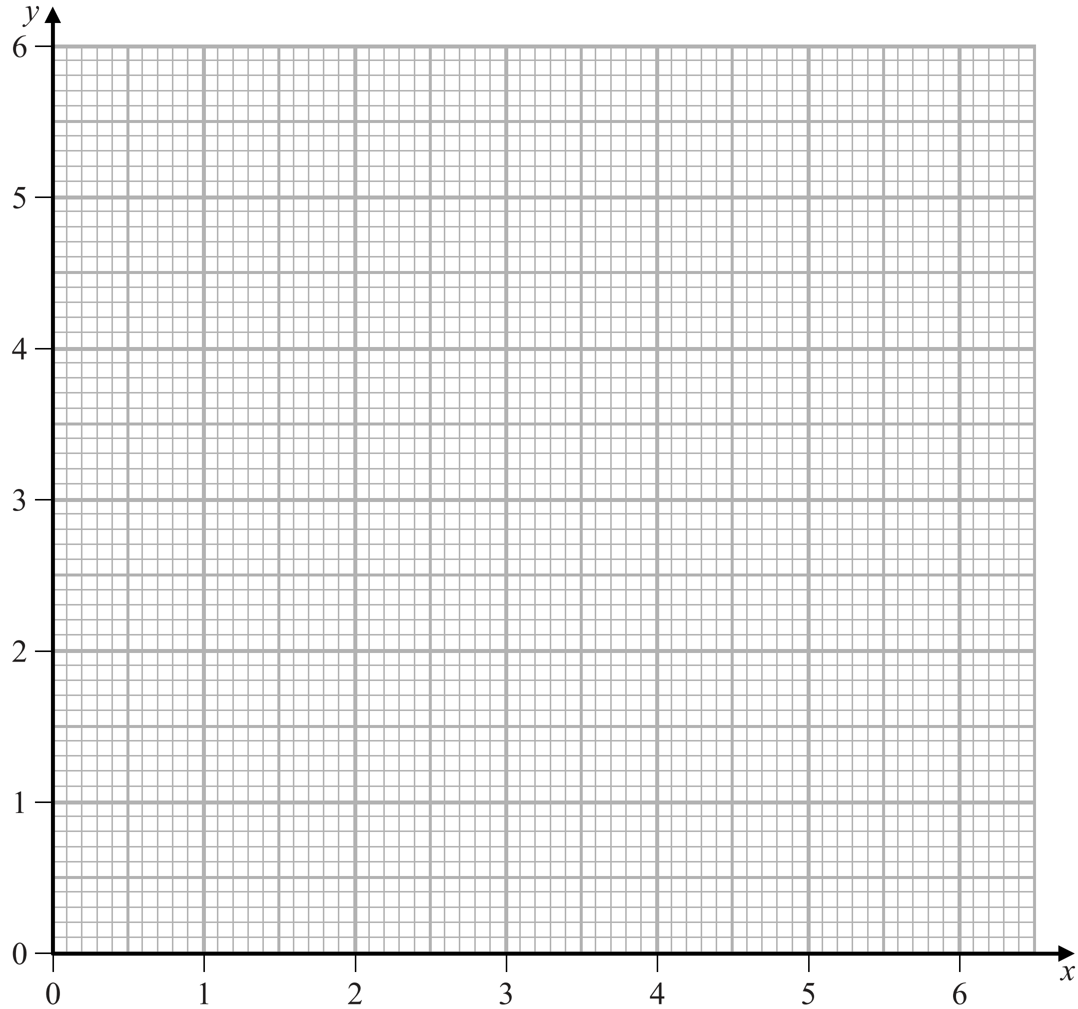
(c) Use your graph to obtain an estimate, to 1 decimal place, for the value of $\ln 10.6$
You **must** show clearly how you have used your graph.

(3)

(d) By drawing a straight line on the grid, obtain estimates, to 1 decimal place, for the roots of the equation $(3x + 1)^2 = e^{(x+1)}$ in the interval $0 \leqslant x \leqslant 6$

(5)

Question 7 continued

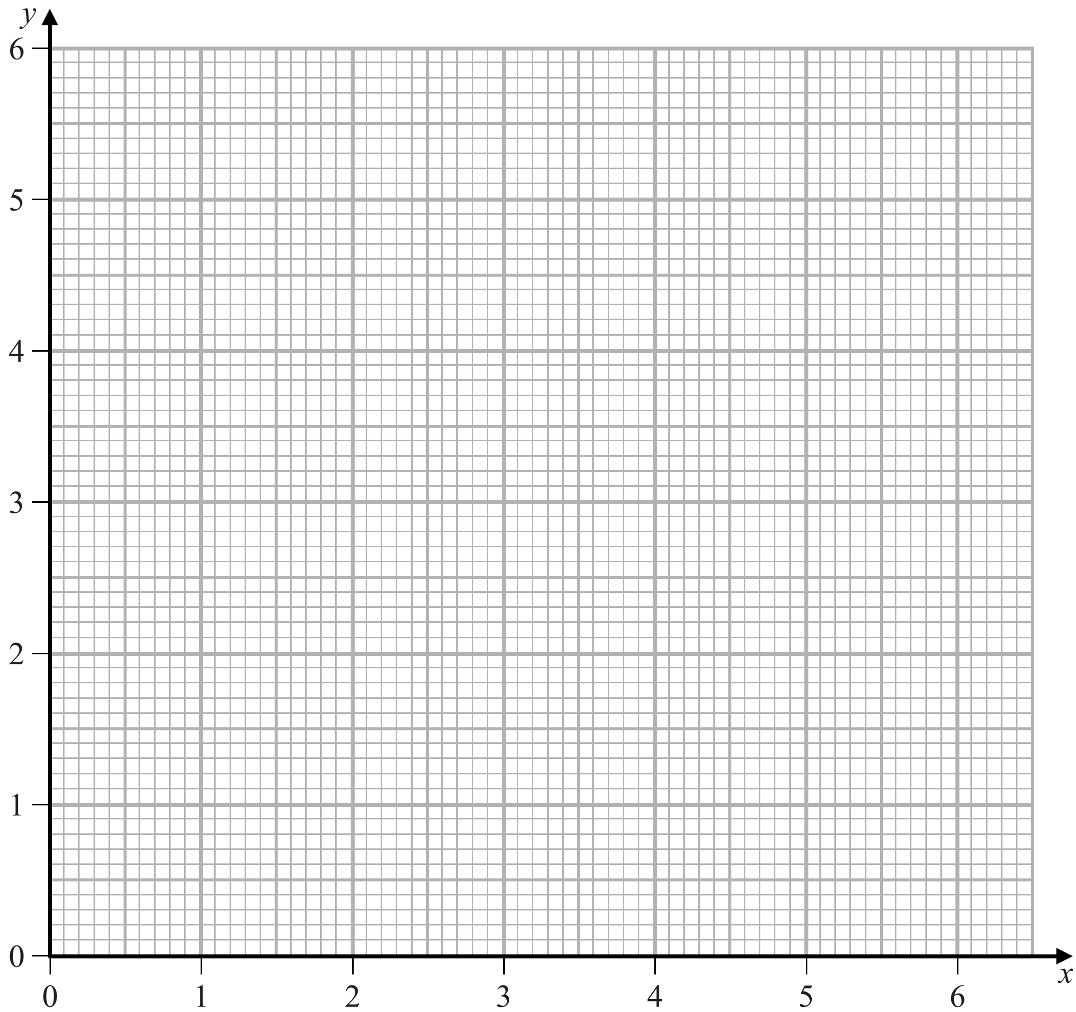


Turn over for a spare grid if you need to redraw your graph.

Question 7 continued

Question 7 continued

Only use this grid if you need to redraw your graph.



(Total for Question 7 is 12 marks)

8 The roots of the equation $3x^2 - 2x - 1 = 0$ are α and β, where $\alpha > \beta$

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$

(3)

(b) show that $\alpha - \beta = \dfrac{4}{3}$

(2)

(c) form a quadratic equation, with integer coefficients, that has roots $\dfrac{\alpha + \beta}{\alpha}$ and $\dfrac{\alpha - \beta}{\beta}$

(6)

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Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9

Diagram **NOT** accurately drawn

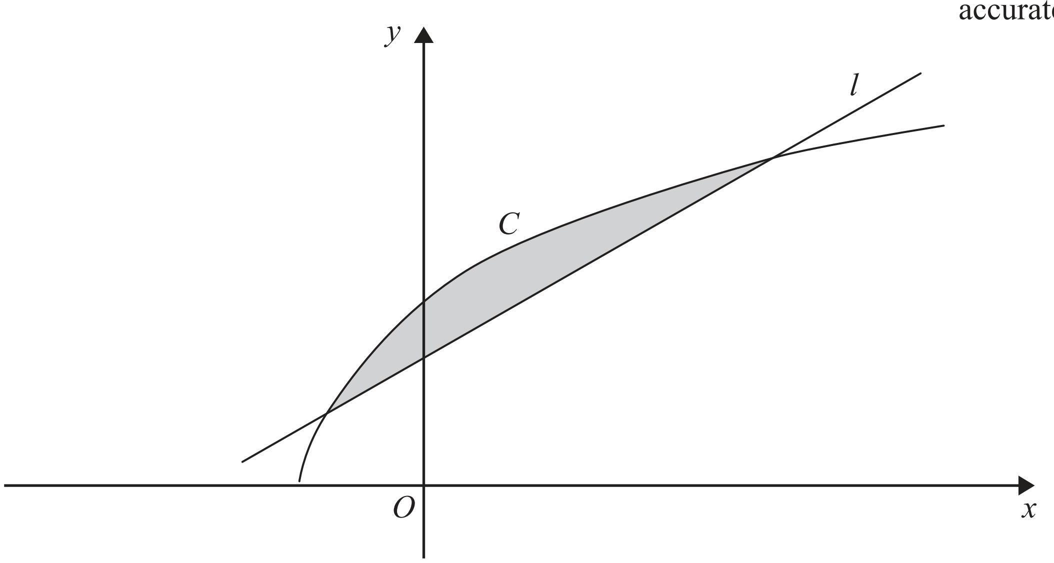

Figure 3

Figure 3 shows part of the curve C with equation $y = (2x + 3)^{\frac{1}{2}}$ and the line l with equation $2y = x + 3$

The line l crosses C at two points.

(a) Find the coordinates of each of these points.

(5)

The finite region bounded by C and l, shown shaded in Figure 3, is rotated through 360° about the x-axis.

(b) Use algebraic integration to find, in terms of π, the volume of the solid generated.

(5)

Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 10 marks)

10 A geometric series has first term a and common ratio r $(r > 0)$
The nth term of the series is U_n

Given that $U_1 + 3U_2 = 8$ and that $U_2 \times U_3 = 4U_5$

(a) find

(i) the value of r

(ii) the value of a

(5)

(b) Hence show that $U_n = \dfrac{2^{n+2}}{3^n}$

(2)

(c) Find the least value of n such that $U_n < 0.05$

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

(a) (i) Using the above identity, show that

$$\cos 2x = 1 - 2\sin^2 x$$

(ii) Hence show that

$$\frac{13\sin x - 2\cos 2x - 10}{4\sin x - 3} = 4 + \sin x$$

(7)

(b) Hence solve, in radians to 3 significant figures, the equation

$$10 + 2\cos\left(2\theta + \frac{\pi}{3}\right) - 13\sin\left(\theta + \frac{\pi}{6}\right) = 2\sin\left(\theta + \frac{\pi}{6}\right) + 8$$

for $\pi \leqslant \theta \leqslant 2\pi$

(5)

(c) Find the exact value of

$$\int_0^{\frac{\pi}{2}} \left(\frac{13\sin x - 2\cos 2x - 10 + 4x\sin x - 3x}{4\sin x - 3}\right) \mathrm{d}x$$

(5)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS